# Arakawa's Computation Device

*Climate science required the invention and mastery of difficult techniques. These had pitfalls, which could lead to controversy. An example of the ingenious technical work and hard-fought debates underlying the main story is Akio Arakawa's invention of a mathematical method that solved a vexing instability in big computer models. For other examples, see the essays on Uses of Radiocarbon Dating and on Temperatures from Fossil Shells.*

Norman Phillips's 1956 experiment was a great success, but after a month it blew up. He had succeeded, for the first time, in constructing a computer model of the general circulation of the atmosphere that produced something looking much like real weather maps. As the calculations stepped forward through time, however, after twenty or so simulated days the pattern of flow began to look strange, and by thirty days the numbers veered off into conditions never seen on Earth.[1]

At first Phillips and his colleagues thought their problem was "round-off error," a difficulty that computer modelers were just beginning to come to grips with. A computer cannot produce numbers with infinite precision (for example, 2.394376113... might be truncated to 2.394). When one round of calculations was fed back into the next round, and this was repeated thousands of times with the numbers truncated each time, tiny discrepancies could add up to a big difference. Eventually the solutions became unrealistic and "exploded." Modelers were working out a variety of solutions to such errors, ways to smooth out artificial bumps in the numbers at one stage or another. It did not take long for Phillips to find that such tricks would not solve his difficulty. There was a more fundamental problem in the way the computer chopped up continuous equations into a grid of numbers. The computer was getting confused by weather patterns that were smaller than the spacing between the grid points.[2]

Phillips's experiment fired the imagination of many, and not least Yale Mintz. He had taken his Ph.D. in meteorology at the University of California, Los Angeles, in 1949, and took up work with the new digital computers. One of his key steps was to recruit Akio Arakawa. Arakawa had graduated from the University of Tokyo shortly after the end of the Second World War only to find job opportunities for scientists severely limited, and took up a career in the growing enterprise of practical meteorology. Like some of his peers, he eventually found a better opportunity in the United States. He was hired to help Mintz express Mintz's ideas in specific mathematical formulas, and to convert the formulas into computer code. Arakawa, however, had ideas of his own. He persuaded Mintz that it would be futile to launch straight into programming

[1] Phillips (1956).

[2] To be precise, "the grid system cannot resolve wave lengths shorter than about 2 grid intervals" and "interprets them incorrectly as long waves." Phillips (1959), p. 501.

when Phillips's experiment had shown there could be instabilities that might wreck the results. Although Mintz was reluctant to delay his project, he gave way and allowed the young man to dig into the fundamental question: how should meteorological equations be represented in numerical calculations?[1]

Following Phillips's lead, Arakawa recognized that the instability was (to make a crude analogy) like the illusory patterns that appear when you look through a window screen at another screen. Where the screens' holes and wires line up, you may see large dark or light blotches ("moiré patterns") that don't really exist beyond the screens. Something like this had turned up in Phillips's equations. Suppose the computer has gone through a complete step of calculating the atmosphere. Among the countless simulated waves sloshing in the simulated atmosphere, many would have wavelengths smaller than the grid spacing. Some of these waves would reach a peak at each grid point. These would look to the computer just like longer waves (in mathematicians' jargon this is "aliasing"). When the computer calculated another step, these particular waves would be selected for a wholly artificial magnification, more at each repetition, until the whole system went awry.

Arakawa sought a way to defeat the problem. The key, he realized, would be a mathematical scheme that described air flow in such a way that certain particular combinations of the numbers representing the flow would remain unchanged. One of these combinations was the kinetic energy, the energy of the wind motion. In the real world, kinetic energy is usually not kept strictly constant. But by finding a way to do calculations that would hold it constant from one step to the next, Arakawa could make sure that no unrealistic spike of wind speed would grow exponentially step by step. He also decided the system should conserve another, more abstruse combination of numbers describing the flow of air, called "enstrophy." For this as for wind speed, calculations at a set of discrete grid points tended to produce an erroneous, unlimited growth. Again the solution would be a mathematical system that held the quantity constant at each step, squelching the spurious magnifications.[2]

Given these basic ideas, Arakawa still had the challenging task of designing equations that would represent the atmosphere in a manner suitable for computer calculation. His path lay through technical mathematics which it is needless to describe here. Making this kind of match between physics and mathematics requires a rare kind of insight, but a flash of inspiration does not bring success without many days of pencil and paper work exploring different avenues. Only after finishing this tough task could Arakawa start the job for which Mintz had brought him to California, coding a program for the computer.

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[1] Here and below I use chiefly Arakawa (2000); see also Johnson and Arakawa (1996), pp. 3217-18.

[2] Ideas on conserving a squared quantity involving energy had been offered by Lorenz (1960). Arakawa, personal communication, 2002, patiently helped me remove some particularly bad errors from this oversimplified account.

This too was no small job, for programming a computer of the day meant writing page after page of instructions, telling the computer to take the number found in register such-and-such and add it to register so-and-so. Then you fed your program into the computer, waited patiently for the printout, and studied the result—which was usually nonsense. There might be a subtle error in the way you had set up the program, but more often you had just hit a wrong key at some point in writing the code. You fixed it, got another printout full of nonsense, and began to hunt down the next problem.[1] Only a perfectionist, which Arakawa was, could persist in such labors for years. Sometimes Mintz gave him a hand, using his intuitive understanding of air flow to suggest where an error lurked, although Mintz never wrote a line of code himself.

Meanwhile, starting in 1962 Arakawa presented his ideas at conferences. (Busy with his programming, and determined to develop general proofs, he did not get around to publishing a journal article until 1966. But the community of modelers was so small and close-knit that by then everyone already knew about the ideas and was using them.)[2] He later recalled that while some of his colleagues were enthusiastic, he had a hard time convincing others that the approach made sense. It seemed intuitively implausible, unlike anything they had seen before and hard to grasp. Some objected that he was misguided to use equations that forced the conservation of a quantity like enstrophy, which nature itself did not conserve. He replied that he was not really dedicated to holding enstrophy constant—it was only a device to make the calculation behave more realistically.

The device worked magnificently. It became a basic component of the UCLA model for the general circulation of the atmosphere. Later this scheme for computing air flow was incorporated in many other models that borrowed ideas and even computer code from the UCLA model, directly or at second hand. Meanwhile, other modelers found a variety of other ways to damp down spurious magnifications and other instabilities. If there is a single best way to build a computer climate model, nobody found it. The community of experts perennially debated the merits and drawbacks of various calculation schemes. Meanwhile Arakawa's basic technique (in technical terms, the numerical conservation of a squared quantity, especially kinetic energy) remained in widespread use. This was only one of many seminal techniques that Arakawa helped introduce to the community of modelers. (For example, a method he published with a collaborator in 1974, describing the behavior of cumulus clouds with a set of parameters that could readily be calculated, became a regular part of other modelers' toolkits.[3]

A notable example of a model that borrowed Arakawa's scheme for computing air flow was the GISS General Circulation Model, begun in the 1970s by a group under James Hansen at NASA's Goddard Institute for Space Studies in New York City. While Arakawa pursued improvements in the elusive mathematics and physics, Hansen wanted to churn out rough results on practical questions as soon as possible, even without full-time access to a supercomputer. To keep the

[1] This passage is based on my own experiences in the 1960s..

[2] Arakawa (1966); Lilly (1997).

[3] Lilly (1997); Arakawa and Schubert (1974).

number of computations manageable, Hansen's group used a grid as much as a thousand kilometers on a side, averaging over all sorts of different weather patterns. After all, it wasn't weather he was interested in, but climate. The model produced a surprisingly realistic simulacrum of atmospheric circulation, including even a jet stream (the real jet stream is often much narrower than a thousand kilometers). That was possible because Arakawa's computation scheme suppressed spurious irregularities in local weather.[1]

Down through the 1990s, Hansen's group used this model for highly influential studies that showed how various factors worked upon climate.[2] They were regularly criticized for using such a coarse grid, in which, for example, the entire contiguous United States was divided into barely a dozen cells. As computers got faster and faster, other modelers minced the planet into ever finer slices. Hansen had to defend his choice of a grid against criticism by the anonymous referees who commented on the proposals he sent to funding agencies and on the papers he submitted for publication. He insisted that making smaller cells would be wasteful, for there were plenty of other things more worth spending his computer time on, factors that did far more to limit the model's validity. He could get away with the coarse geographical resolution thanks to the power of Arakawa's computation device.[3]

**Related:**
General Circulation Models of the Atmosphere

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[1] Hansen et al. (1983).

[2] E.g., Hansen et al. (1992); Hansen et al. (1984).

[3] Hansen et al. (1983); Hansen, interview by Weart, Oct. 2000, AIP and Hansen et al. (2000a).